

Color symmetrical superconductivity in a schematic nuclear quark model.

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Abstract

In this paper, the phenomenon of color superconductivity is analyzed on the basis of a novel BCS-type formalism and in the context of a schematic QCD inspired quark model, having in mind the description of color symmetrical superconducting states. The QCD-like model has color $SU(3)$ symmetry and is constructed in terms of the generators of the $su(4)$ algebra, becoming exactly solvable. The physical properties of the BCS vacuum (average numbers of quarks of different colors) remain unchanged under an arbitrary color rotation. In the usual approach to color superconductivity, the pairing correlations affect only the quasi-particle states of two colors, the single particle states of the third color remaining unaffected by the pairing correlations. In the theory of color symmetrical superconductivity here proposed, the pairing correlations affect symmetrically the quasi-particle states of the three colors so that vanishing net color-charge is automatically insured. It is found that the ground-state energy of the color symmetrical sector of this exactly solvable model is well approximated by the average energy of the color symmetrical superconducting state proposed here.

1 Introduction

At present, it is generally accepted that QCD matter at high densities exhibits color superconductivity induced by the familiar phenomenon of Cooper instability [1]. For a recent review, see [2]. That color superconductivity breaks color $SU(3)$ symmetry down to the $SU(2)$ level is a familiar statement, which, however, may require further specification. Quarks are free in the deconfined phase, but the deconfined phase itself is believed to be a color singlet. For instance, in Refs. [3, 4] it is argued that in QCD the superconducting phase is automatically color symmetrical.¹ Our aim is to develop a version of the BCS theory which is appropriate to describe a

¹The expressions “color singlet”, “color symmetrical” and “color neutral”, are here used as synonymous, and refer to states belonging to one dimensional representation of color $SU(3)$.

color singlet superconducting phase. The physical properties of the BCS vacuum which is here constructed (average numbers of quarks of different colors) remain unchanged under an arbitrary color rotation. The problem of reconciling color superconductivity with color symmetry has been addressed by some authors [5, 6]. In [5], the authors resort to a rather involved projection technique to extract color symmetrical states out of BCS states which violate color symmetry. In [6], color symmetry is imposed by requiring that the average or expectation value of some of the eight color generators vanish, that is, color neutrality is implemented in the average. The approach followed in [5] suffers from an obvious deficiency which is known to be associated with the Peierls-Yoccoz projection method, namely, the reliability of the result obtained depends strongly on the choice of the meanfield state to which the method is applied (see [7], pp. 460-462). That meanfield state must to be properly chosen, that is, it has to be chosen in such a way that you may reasonably expect the components with the desired symmetry to have a dominant contribution. Otherwise you end up by extracting a component which, of course, has the desired symmetry but which most probably has nothing to do with the state you are looking for. For instance, suppose you are studying a deformed nucleus and wish to describe the lowest energy state with angular momentum J by applying the Peierls-Yoccoz projection method to a meanfield state vector. You may easily extract a state with the desired angular momentum which, however, most probably will lead to the wrong moment of inertia. The projection method is reliable only if the mean field state has been constructed according to very precise rules, following a procedure analogous to the one we will describe in the sequel. Thus, the following development is of interest even if you have in mind using the projection method, as in [5].

A BCS state $|\Phi\rangle$ describes a physical state with zero net color charge if $N_1 = N_2 = N_3$, where N_i denotes the average number of quarks of color i . This means that

$$\langle\Phi|S_{11}|\Phi\rangle = \langle\Phi|S_{22}|\Phi\rangle = \langle\Phi|S_{33}|\Phi\rangle. \quad (1)$$

Here, S_{kl} denote the color $U(3)$ generator which will be defined in the next section. However, we will show that the requirement (1), which is implemented in [6], is not sufficient to insure that $|\Phi\rangle$ describes a color singlet, which is the condition for physical acceptability. A stronger condition must then be imposed. Indeed, the $SU(3)$ symmetry, being a gauge symmetry, cannot be broken, according to the discussion in [3, 4]. Therefore, color rotated BCS states must be equivalent in the sense of the physics they describe. Let U_c denote an arbitrary color rotation, i.e., $U_c = \exp \sum_{k,l=1}^3 ix_{kl}S_{kl}$, $x_{kl} = x_{lk}^*$. The BCS state $|\Phi\rangle$ must be equivalent to the state $U_c|\Phi\rangle$, for any U_c , as far as expectation values of physical observables are concerned. Thus, the condition (1) must be replaced by

$$\langle\Phi|U_c^\dagger S_{11} U_c |\Phi\rangle = \langle\Phi|U_c^\dagger S_{22} U_c |\Phi\rangle = \langle\Phi|U_c^\dagger S_{33} U_c |\Phi\rangle,$$

for an arbitrary U_c , and this implies

$$\langle\Phi|S_{11}|\Phi\rangle = \langle\Phi|S_{22}|\Phi\rangle = \langle\Phi|S_{33}|\Phi\rangle, \quad \langle\Phi|S_{kl}|\Phi\rangle = 0, \quad \text{for } k \neq l. \quad (2)$$

This is the condition the BCS state $|\Phi\rangle$ must necessarily satisfy in order to be physically meaningful. If only the condition (1) is implemented, as in [6], and not the condition (2), the BCS state $|\Phi\rangle$ is, in general, not equivalent to the state $U_c|\Phi\rangle$, so that it describes a state belonging to a representation of $SU(3)$ other than the

singlet one, which is physically unacceptable. In [8], it has already been suggested that the condition (2) should be satisfied if we wish that the BCS state $|\Phi\rangle$ describes a color singlet state. Here, we show how this condition may be easily implemented.

We approach this problem by constructing the BCS state through an appropriate generalization of the Bogoliubov transformation that treats all colors on the same footing. For simplicity, the present derivation is performed in the framework of a standard nuclear quark model exemplified by the Bonn model which was proposed by H.R. Petry *et al.* in 1985 [9], but the scope of the formalism here obtained is quite general. This model is defined through a specific color dependent pairing interaction which is expressed in terms of certain generators of $SU(4)$ and is invariant under $SU(3)$. Due to this important symmetry, that model is usually regarded as a model for the formation of color symmetrical triplets. In ref. [8], exact solutions have been presented to the equations of the model, it being shown that, in general, its groundstate is not color symmetrical, although it admits an important color symmetrical sector. Using a representation of $su(4)$ of the Schwinger type which is formulated in terms of appropriate boson operators and is due to Yamamura *et al.* [10], it has been possible to characterize the referred color symmetrical sector [11]. In ref. [8], it has been shown that BCS states describe adequately the groundstate of the model. In this note we focus on the description of the color symmetrical sector of the model.

2 A schematic $su(3)$ pairing model

The quark model proposed by H.R. Petry *et al.* [9] is defined by the Hamiltonian

$$H = G \sum_{j=1}^3 A_j^\dagger A_j, \quad (3)$$

where

$$A_1^\dagger = \sum_{m>0} (c_{2m}^\dagger c_{3\overline{m}}^\dagger + c_{2\overline{m}}^\dagger c_{3m}^\dagger), \quad \overline{\overline{m}} = m, \quad (4)$$

$G < 0$ is the coupling constant and the expressions for A_2^\dagger, A_3^\dagger , are obtained by circular permutation of the indices 1, 2, 3. In eq. (4), c_{im}^\dagger are quark creation operators and the indices i and m denote, respectively, the color and the remaining single particle quantum numbers. By \overline{m} we mean the state obtained from m by time reversal. We remark, in passing, that this interaction is of the same type as the one used to describe color superconducting quark matter [1]. The present model is an effective QCD model in the limit of asymptotic freedom of vanishing nuclear forces. In a future study we shall include the barions as a three-body force.

Color superconductivity has been applied in [8] to the description of the ground-state of the Bonn model, which in general is not color symmetric. Indeed, although H has $SU(3)$ symmetry, its eigenstates are not necessarily invariant under color $SU(3)$ rotations, that is, they are not necessarily color singlets. The study of the color symmetrical sector is particularly interesting. The generators of color $U(3)$ read

$$S_{kl} = \sum_m c_{km}^\dagger c_{lm} = \sum_{m>0} (c_{km}^\dagger c_{lm} + c_{k\overline{m}}^\dagger c_{l\overline{m}}).$$

A state $|\Phi\rangle$ is a color singlet if it satisfies the following condition

$$S_{kl}|\Phi\rangle = 0, \quad k \neq l, \quad S_{kk}|\Phi\rangle = \lambda|\Phi\rangle, \quad k = 1, 2, 3. \quad (5)$$

A BCS state can not satisfy (5), so that, strictly speaking, it can not be a color singlet, but it will describe a color singlet if, and only if, it satisfies (2), which amounts to satisfying (5) in the average. It has been remarked in [8] that the description of the color symmetrical sector of the model requires a modified BCS state, defined as the vacuum of the quasi-particles of an appropriate generalized Bogoliubov transformation.

3 Color symmetric BCS state

The most general case BCS state which, in the average, is color neutral, reads

$$|\Phi\rangle = \exp \sum_{j=0}^3 \left(K \sum_{0 < m \leq \Omega'} A_{jm}^\dagger + \tilde{K} \sum_{\Omega' < m \leq \Omega} A_{jm} \right) |0_{\Omega'}\rangle,$$

where

$$|0_{\Omega'}\rangle = \left(\prod_{j=1}^3 \prod_{\Omega' < m \leq \Omega} c_{jm}^\dagger c_{j\bar{m}}^\dagger \right) |0\rangle,$$

and

$$A_{1m}^\dagger = (c_{2m}^\dagger c_{3\bar{m}}^\dagger + c_{2\bar{m}}^\dagger c_{3m}^\dagger).$$

This state is color neutral in the sense of vanishing net color charge. The expressions for $A_{2m}^\dagger, A_{3m}^\dagger$, are obtained by circular permutation of the indices 1, 2, 3. The parameters K, \tilde{K} are real. We denote by 2Ω the level degeneracy for a fixed color, that is, the totality of eigenstates pertaining to all quantum numbers beyond color. If $\Omega' = \Omega$, the quark number N satisfies $0 \leq N \leq 4\Omega$. If $\Omega' = 0$, the quark number N satisfies $4\Omega \leq N \leq 6\Omega$. The state vector $|\Phi\rangle$ has obviously zero net color charge, but it is not color symmetrical. This is because $[S_{12}, (A_1^\dagger + A_2^\dagger + A_3^\dagger)] = -A_2^\dagger \neq 0$. However, K, \tilde{K} may be chosen so that $|\Phi\rangle$ is color symmetrical in the average, that is, so that (1) is satisfied. We observe that

$$\begin{aligned} c_{1m}|\Phi\rangle &= K(c_{2\bar{m}}^\dagger - c_{3\bar{m}}^\dagger)|\Phi\rangle, & c_{1\bar{m}}|\Phi\rangle &= K(c_{2m}^\dagger - c_{3m}^\dagger)|\Phi\rangle, & 0 < m \leq \Omega', \\ c_{1m}^\dagger|\Phi\rangle &= -\tilde{K}(c_{2\bar{m}} - c_{3\bar{m}})|\Phi\rangle, & c_{1\bar{m}}^\dagger|\Phi\rangle &= -\tilde{K}(c_{2m} - c_{3m})|\Phi\rangle, & \Omega' < m \leq \Omega. \end{aligned} \quad (6)$$

These relations are crucial. They are straightforward consequences of the commutation relations

$$\begin{aligned} \left[c_{1p}, \left(K \sum_{0 < m \leq \Omega'} A_{jm}^\dagger + \tilde{K} \sum_{\Omega' < m \leq \Omega} A_{jm} \right) \right] &= K(c_{2\bar{p}}^\dagger - c_{3\bar{p}}^\dagger), & 0 < p \leq \Omega', \\ \left[c_{1p}^\dagger, \left(K \sum_{0 < m \leq \Omega'} A_{jm}^\dagger + \tilde{K} \sum_{\Omega' < m \leq \Omega} A_{jm} \right) \right] &= -\tilde{K}(c_{2\bar{p}} - c_{3\bar{p}}), & \Omega' < p \leq \Omega. \end{aligned}$$

From (6) it follows that the BCS vacuum $|\Phi\rangle$ is annihilated by the operators

$$\begin{aligned} d_{1m} &= c_{1m} - K(c_{2\bar{m}}^\dagger - c_{3\bar{m}}^\dagger), & d_{1\bar{m}} &= c_{1\bar{m}} - K(c_{2m}^\dagger - c_{3m}^\dagger), & 0 < m \leq \Omega', \\ d_{1m} &= c_{1m}^\dagger + \tilde{K}(c_{2\bar{m}} - c_{3\bar{m}}), & d_{1\bar{m}} &= c_{1\bar{m}}^\dagger + \tilde{K}(c_{2m} - c_{3m}), & \Omega' < m \leq \Omega. \end{aligned} \quad (7)$$

The expressions for d_{2m} , d_{3m} , $d_{2\bar{m}}$, $d_{3\bar{m}}$, are obtained by circular permutation of the indices 1,2,3. These operators characterize the so-called Bogoliubov quasi-particles. The transformation in eq. (7) is not canonical, since $\{d_{im}, d_{jm}^\dagger\} \neq \delta_{ij}$, but the corresponding canonical transformation, which is not needed for the present purpose, may be easily obtained.²

We introduce the notation $\langle W \rangle = \langle \Phi | W | \Phi \rangle / \langle \Phi | \Phi \rangle$. We observe that the contractions $\langle c_{im}^\dagger c_{jm} \rangle$, $i \neq j$, are independent of i , j . Similarly the contractions $\langle c_{j\bar{m}}^\dagger c_{jm} \rangle$, are independent of j .

We easily find

$$\langle c_{im}^\dagger c_{jm} \rangle = \langle c_{i\bar{m}}^\dagger c_{j\bar{m}} \rangle = -\frac{K^2}{1+3K^2}, \quad i \neq j, \quad \langle c_{jm}^\dagger c_{jm} \rangle = \langle c_{j\bar{m}}^\dagger c_{j\bar{m}} \rangle = \frac{2K^2}{1+3K^2},$$

if $0 < m \leq \Omega'$.

On the other hand,

$$\langle c_{im}^\dagger c_{jm} \rangle = \langle c_{i\bar{m}}^\dagger c_{j\bar{m}} \rangle = \frac{\tilde{K}^2}{1+3\tilde{K}^2}, \quad i \neq j, \quad \langle c_{jm}^\dagger c_{jm} \rangle = \langle c_{j\bar{m}}^\dagger c_{j\bar{m}} \rangle = 1 - \frac{2\tilde{K}^2}{1+3\tilde{K}^2},$$

if $\Omega' < m \leq \Omega$.

These results are obtained as follows. For $0 < m \leq \Omega'$, we have

$$X := \langle c_{1m}^\dagger c_{2m} \rangle = -K^2 - K^2(-\langle c_{3\bar{m}}^\dagger c_{3\bar{m}} \rangle - \langle c_{2\bar{m}}^\dagger c_{1\bar{m}} \rangle + \langle c_{3\bar{m}}^\dagger c_{1\bar{m}} \rangle + \langle c_{2\bar{m}}^\dagger c_{3\bar{m}} \rangle)$$

$$N := \langle c_{1m}^\dagger c_{1m} \rangle = 2K^2 - K^2(\langle c_{3\bar{m}}^\dagger c_{3\bar{m}} \rangle + \langle c_{2\bar{m}}^\dagger c_{2\bar{m}} \rangle - \langle c_{3\bar{m}}^\dagger c_{2\bar{m}} \rangle - \langle c_{2\bar{m}}^\dagger c_{3\bar{m}} \rangle),$$

implying

$$X = -K^2 + K^2(N - X), \quad N = 2K^2 - 2K^2(N - X),$$

which leads to $X = -K^2/(1+3K^2)$, $N = 2K^2/(1+3K^2)$. The corresponding expressions for $\Omega' < m \leq \Omega$ are similarly obtained. We find

$$\langle S_{ij} \rangle = -2\Omega' \frac{K^2}{1+3K^2} + 2(\Omega - \Omega') \frac{\tilde{K}^2}{1+3\tilde{K}^2}, \quad i \neq j.$$

By conveniently choosing K , \tilde{K} , we may insure that the condition for the BCS vacuum $|\Phi\rangle$ to be a color singlet, namely

$$\langle \Phi | S_{ij} | \Phi \rangle = 0, \quad \text{for } i \neq j, \quad (8)$$

is satisfied. The state $|\Phi\rangle$ satisfies automatically (1). However, (1) remains valid when we replace $|\Phi\rangle$ by $U_c|\Phi\rangle$, for an arbitrary color rotation U_c , only if (8) is further imposed.

Next we compute the contractions $\langle c_{2m} c_{1\bar{m}} \rangle = \langle c_{3m} c_{2\bar{m}} \rangle = \langle c_{1m} c_{3\bar{m}} \rangle = \langle c_{2\bar{m}} c_{1m} \rangle = \langle c_{3\bar{m}} c_{2m} \rangle = \langle c_{1\bar{m}} c_{3m} \rangle =: D_m$, where D_m is real. For $0 < m \leq \Omega'$ we have

$$\begin{aligned} \langle c_{2\bar{m}} c_{1m} \rangle &= K - K^2 \left(\langle c_{2\bar{m}}^\dagger c_{3m}^\dagger \rangle + \langle c_{3\bar{m}}^\dagger c_{1m}^\dagger \rangle + \langle c_{1\bar{m}}^\dagger c_{2m}^\dagger \rangle - \langle c_{3\bar{m}}^\dagger c_{3m}^\dagger \rangle \right), \\ \langle c_{1\bar{m}} c_{1m} \rangle &= -K^2 \left(\langle c_{2\bar{m}}^\dagger c_{2m}^\dagger \rangle + \langle c_{3\bar{m}}^\dagger c_{3m}^\dagger \rangle - \langle c_{3\bar{m}}^\dagger c_{2m}^\dagger \rangle - \langle c_{2\bar{m}}^\dagger c_{3m}^\dagger \rangle \right), \\ &= -K^2 \left(\langle c_{2\bar{m}}^\dagger c_{2m}^\dagger \rangle + \langle c_{3\bar{m}}^\dagger c_{3m}^\dagger \rangle \right), \end{aligned}$$

²A generalized Bogoliubov transformation has been proposed in eq. (25) of [8]. However, the associated BCS state does not satisfy (2). The transformation (7) replaces eq. (25) of [8].

which imply

$$D_m = K - 3K^2 D_m + K^2 P_m, \quad P_m = 2K^2 P_m,$$

where $P_m := \langle c_{1m} c_{1\bar{m}} \rangle = \langle c_{2m} c_{2\bar{m}} \rangle = \langle c_{3m} c_{3\bar{m}} \rangle$, is also real. The procedure for $\Omega' < m \leq \Omega$, is analogous. Finally, we find $P_m = 0$ and

$$D_m = \frac{K}{1 + 3K^2}, \quad 0 < m \leq \Omega'; \quad D_m = \frac{\tilde{K}}{1 + 3\tilde{K}^2}, \quad \Omega' < m \leq \Omega.$$

We are now able to compute the energy expectation value

$$\frac{\mathcal{E}}{G} = \sum_{j=1}^3 \frac{\langle \Phi | A_j^\dagger A_j | \Phi \rangle}{\langle \Phi | \Phi \rangle}.$$

3.1 Computation of the energy expectation value

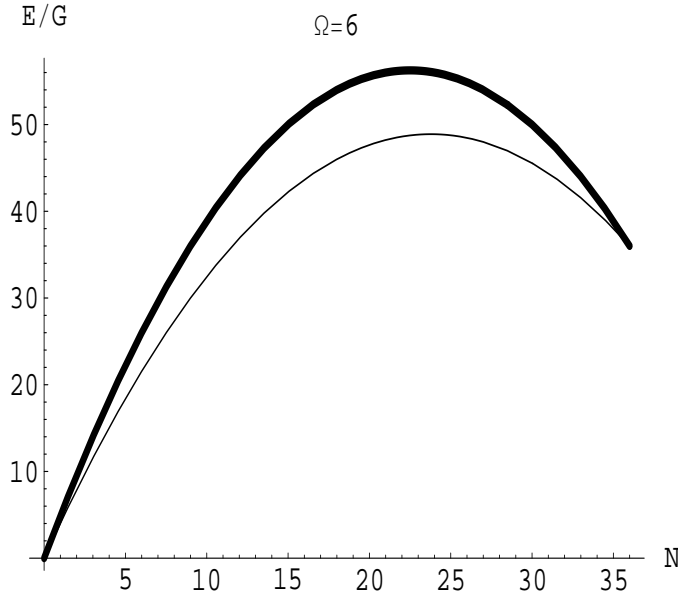


Figure 1: Groundstate energy of the color symmetrical sector versus the quark number, for $\Omega = 6$. Thick line: exact result according to (10); thin line: color symmetrical BCS estimate according to (9). Since $G < 0$, the maximum of E/G corresponds to the minimum of E .

Let $p = \Omega'/\Omega$, $q = 1 - p = (\Omega - \Omega')/\Omega$. Define θ , $\tilde{\theta}$ such that $\sqrt{3}K/\sqrt{1 + 3K^2} = \sin \theta$, $1/\sqrt{1 + 3K^2} = \cos \theta$, $\sqrt{3}\tilde{K}/\sqrt{1 + 3\tilde{K}^2} = \sin \tilde{\theta}$, $1/\sqrt{1 + 3\tilde{K}^2} = \cos \tilde{\theta}$. Then, the color symmetry constraint reads

$$p \cos 2\theta - q \cos 2\tilde{\theta} = p - q.$$

The main contribution to the energy expectation value comes from the square of the following expectation values, which involve contractions of the form $\langle cc \rangle$,

$$\langle A_1 \rangle = \langle A_2 \rangle = \langle A_3 \rangle = \frac{\Omega}{\sqrt{3}}(p \sin 2\theta + q \sin 2\tilde{\theta}).$$

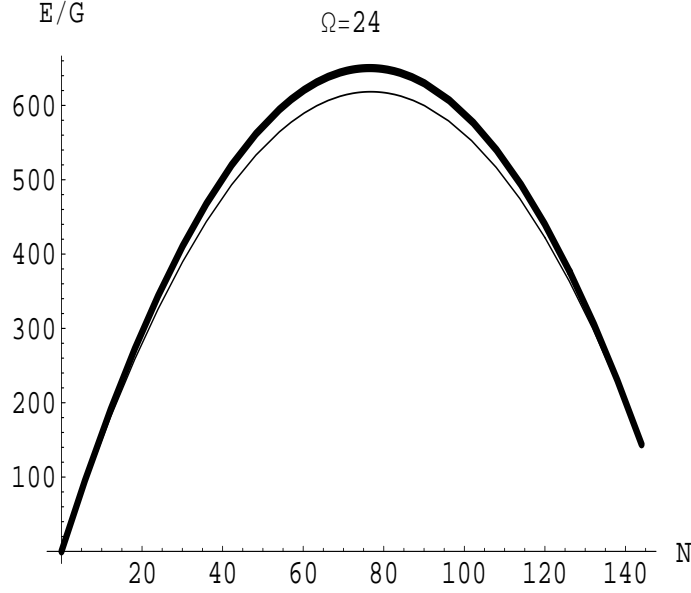


Figure 2: Groundstate energy of the color symmetrical sector versus the quark number, for $\Omega = 24$. Thick line: exact result according to (10); thin line: color symmetrical BCS estimate according to (9).

The corresponding constrained extremum occurs for

$$\cos 2\theta = -\cos 2\tilde{\theta} = p - q, \quad \sin 2\theta = \sin 2\tilde{\theta} = \sqrt{1 - (p - q)^2},$$

so that, in the leading order, $\mathcal{E}/G \approx \Omega^2(1 - (p - q)^2)$. In terms of the variables $\theta, \tilde{\theta}$, the number of quarks reads $N = 6\Omega[p(1 - \cos 2\theta)/3 + q(1 - (1 - \cos 2\tilde{\theta})/3)]$. At the extremum, $N = 6\Omega q$, and $\mathcal{E}/G \approx \Omega^2[1 - (1 - N/(3\Omega))^2]$. To complete the calculation of \mathcal{E}/G we must add the small corrections coming from the neglected contractions of the form $\langle c^\dagger c \rangle$.

In terms of $\theta, \tilde{\theta}$, we have

$$\langle c_{jm}^\dagger c_{jm} \rangle = \langle c_{j\bar{m}}^\dagger c_{j\bar{m}} \rangle = \frac{1}{3}(1 - \cos 2\theta), \quad 0 < m \leq \Omega',$$

$$\langle c_{jm}^\dagger c_{jm} \rangle = \langle c_{j\bar{m}}^\dagger c_{j\bar{m}} \rangle = 1 - \frac{1}{3}(1 - \cos 2\tilde{\theta}), \quad \Omega' < m \leq \Omega.$$

At the extremum,

$$\langle c_{jm}^\dagger c_{jm} \rangle = \langle c_{j\bar{m}}^\dagger c_{j\bar{m}} \rangle = \frac{2}{3} q, \quad 0 < m \leq \Omega',$$

$$\langle c_{jm}^\dagger c_{jm} \rangle = \langle c_{j\bar{m}}^\dagger c_{j\bar{m}} \rangle = \frac{1}{3}(1 + 2q), \quad \Omega' < m \leq \Omega.$$

The contribution, mentioned above, of the neglected contractions of the form $\langle c^\dagger c \rangle$, to \mathcal{E}/G , is

$$6\Omega \left[p \left(\frac{2}{3} q \right)^2 + \left(\frac{1}{3}(1 + 2q) \right)^2 \right] = \frac{2\Omega q}{3}(1 + 8q) = \frac{N}{9} \left(1 + \frac{4N}{3\Omega} \right).$$

Finally, the groundstate energy of the color symmetrical super-conducting phase reads,

$$\frac{\mathcal{E}}{G} = \frac{N}{9} \left(6\Omega - N + 1 + \frac{4N}{3\Omega} \right), \quad 0 \leq N \leq 6\Omega. \quad (9)$$

Although eq. (9) is close to eq. (29) of ref. [8], showing the same qualitative behavior, this agreement is, to some extent, accidental.

Using the Schwinger type representation of $su(4)$, formulated in terms of appropriate boson operators, which was developed by Yamamura *et al.* [10], the color symmetrical sector of the Bonn model has been characterized in [11]. There, the exact groundstate energy of the color symmetrical sector was found to read

$$\frac{\mathcal{E}}{G} = \frac{N}{3} \left(2\Omega + 3 - \frac{N}{3} \right), \quad 0 \leq N \leq 6\Omega. \quad (10)$$

It is interesting to compare eq. (9) with eq. (10), because the comparison shows that the approximate result of (9) is in reasonable agreement with the exact result of (10).

4 Conclusions

We have constructed a BCS-type formalism, based on a conveniently generalized Bogoliubov transformation, which is appropriate to describe color symmetrical superconducting states of quark matter. It is interesting to compare eq. (9), showing the BCS estimate of the groundstate energy of the color symmetric sector, and eq. (10), showing the exact groundstate energy of the same sector. This is done, in fig. 1, for $\Omega = 6$ and in fig. 2, for $\Omega = 24$. It is clearly seen that the color symmetric BCS result becomes closer to the exact one while Ω increases.

It should be emphasized that the present approach automatically ensures vanishing net color charge, in the average.

A word is in order concerning the mechanism of color-flavor-locking (CFL) which was introduced in [1] and does lead to color symmetric superconductivity, in the average. However, CFL is only meaningful if the three flavors u, d, s are equally relevant. The new form of color symmetrical superconductivity which is proposed here is, of course, distinct from CFL, and is also meaningful when only the flavors u, d are relevant. The CFL property considered in ref. [1] is destroyed by an arbitrary infinitesimal color rotation, unless eq. (1) is implemented.

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